B. Optional Dual Polarization- ZDR, PHIDP, KDP, LDR, ...

B.1 Overview of Dual Polarization

Polarization measurements can provide additional information that can be used to determine more accurate measurements of rainfall or, in some cases, infer particle type such as hail or graupel. The fundamental basis for polarization is that raindrops, particularly larger ones, are not spherical — they are oblate (flattened) such that the horizontal axis is longer than the vertical axis. This means that raindrops will respond differently, for example, to vertical and horizontal polarization of the electric field vector. Because of this, and for technical reasons, most polarization radars use horizontal and vertical polarization. For a review of polarization techniques and variables, please refer to Doviak and Zrnic (1993) section 8.5.

Fundamentally a polarization radar measures amplitude and phase in the same manner as a conventional radar. The new information is that the amplitude and phase can be measured at more than one polarization. The differences in amplitudes and phases measured at different polarizations contain information on the presence or absence of non-spherical scatterers such as large flattened drops. For convenience, some of the basic polarization variables are described below:

ZDR: Differential Reflectivity

In the case of amplitude (power) measurements, the larger horizontal axis of drops causes the power measured at horizontal polarization (of the electric field) to be larger than the power measured at vertical polarization. The ratio of the reflectivity factors Z_H/Z_V expressed in dB is given the name ZDR or differential reflectivity. It is generally positive in rain (i.e., >1) and is usually less than about 5 dB. When the rainfall rate is large, there are typically more large drops so that ZDR is larger. Low ZDR and high dBZ indicates the presence of hail which is perhaps tumbling with no preferred orientation. ZDR, because it is a ratio of powers, is not sensitive to the radar calibration as long as the overall gain of the H and V channels is the same (or calibrated).

PhiDP and KDP: Differential Phase and Specific Differential Phase

In the case of phase measurement, the speed of propagation is also affected by the asymmetry of the larger drops. Because of the longer dimension of the horizontal axis of drops, the medium is effectively more dense for horizontal than for vertical polarization so that the speed of light is reduced for horizontal polarization. This causes the horizontal wavelength to be slightly compressed (more phase cycles per unit distance) in comparison with the vertical wavelength which leads to a phase difference between horizontal and vertical. The phase difference $\Phi_H - \Phi_V$ is called Φ_{DP} differential phase shift. Φ_{DP} increases with range since the phase shifts faster (more frequency cycles per unit distance) for the compressed horizontal microwaves as compared to the faster vertical microwaves. The range derivative of the differential phase, i.e., the change of phase per unit distance, is called K_{DP} or the specific differential phase. K_{DP} is almost directly proportional to the rainfall rate so that it has the potential for improving precipitation rate measurements as compared to traditional Z–R relationship measurements which can be highly inaccurate.

LDR: Linear Depolarization Ratio

Some advanced polarization radars can transmit at one polarization and receive simultaneously in two channels, usually the co–polarized and cross–polarized components. For example, when transmitting horizontal, both horizontal (co–polarized) and vertical (cross–polarized) are received by two separate channels. In the case of vertical or horizontal, the ratio of the power Z_{cross} / Z_{co} is called the linear depolarization ratio or LDR. The amount of incident radiation that is depolarized by a particle depends on the particle shape and orientation (e.g., canting angle with respect to horizontal). Perfectly spherical particles do not depolarize either horizontal or vertical polarization so that LDR is zero. Particles that are wet, tumbling and irregularly shaped will give larger LDR values. Therefore, LDR values in rain tend to be small, e.g., less than –25dB. Larger values of LDR can occur in the bright band or in the presence of hail.

A radar and antenna system must be optimized to measure LDR by assuring that the antenna, feed and supporting struts and radome are not themselves depolarizing the transmitted and received radiation. This is called "cross–pol isolation". The integrated cross–pol isolation of the antenna pattern must be better than about 30 dB for LDR measurement since –20 dB is a large LDR.

[RHOHV, PHIDP] [RHOH, PHIH] [RHOV, PHIV]: Correlation Variables

There are several correlation functions that can be calculated depending on the capabilities of the radar. These are generally complex having both an amplitude and phase. These are all normalized so that a perfect correlation magnitude is 1 and perfectly decorrelated is 0.

RHOHV and PHIDP are the magnitude and phase of the correlation between the horizontal and vertical co-polarized channels. These are available on H/V switching systems or on systems that transmit simultaneous H and V. As discussed in a preceding paragraph, PHIDP can be used to infer precipitation rate. RHOHV in rain is typically very close to 1 (0.98). RHOHV values can be reduced in the case of irregularly shaped, randomly oriented, wet tumbling particles. Thus RHOHV has information on the particle type.

RHOH and PHIH are the magnitude and phase of the correlation between the co-polarized and cross-polar channels for H transmission and simultaneous H and V reception. RHOV and PHIV denote the cross–channel correlation magnitude and phase for vertical transmission. These are available on dual-channel receiver with transmit either fixed or alternating. The information content of the cross-pol correlations is the topic of current research.

B.2 Radar System Considerations

A polarization radar is characterized by how it transmits and how it receives. For simplicity we will assume that the radar uses horizontal and/or vertical polarization. However, other polarization pairs could be used (e.g., right and left circular polarization).

Transmit Modes

- **Fixed** (horizontal or vertical)- this can be controlled by a switch or the radar can be simply fixed to transmit a single polarization. If a switch is used, it can be a simple slow waveguide switch rather than a fast switch (pulse-to-pulse).
- **Alternating** (horizontal and vertical)- in this case the radar alternates pulse-to-pulse between horizontal and vertical. A high-power fast switch is used to switch the polarization between the two channels.
- **Simultaneous** (horizontal and vertical)- horizontal and vertical are transmitted simultaneously.

Receive Modes

- **Single-channel receiver**- used only for alternating transmission. The receiver typically receives the co–polarized radiation (transmit H and receive H then transmit V and receive V).
- **Dual-channel receiver**- receives two channels (H and V) simultaneously.

The table below summarizes the various transmit and receive cases and the polarization variables that are available for each. Note that standard parameters are available for all cases (dBT, dBZ, V and W). The RVP8 supports all of these cases.

Table B–1: Transmitter Types

	Transmitter Type			
Receiver Type	**Fixed H**	**Fixed V**	**Alternating H&V**	**Simultaneous H+V**
Single-Channel	Conventional Radar	Conventional Radar	ZDR RHOHV PHIDP and KDP	Not applicable
Dual-Channel	LDRH RHOH PHIH	LDRV RHOV PHIV	LDRH LDRV RHOH RHOV PHIH PHIV ZDR RHOHV PHIDP and KDP	ZDR RHOHV PHIDP and KDP (*STAR mode*)

The fixed single -channel cases are conventional radars rather than polarization radars. The case of simultaneous H+V transmission and a single radar does not make physical sense. The other cases provide various polarization measurements. The fixed dual-channel cases allow the cross-polarization LDR and the co-pol/cross-pol correlation amplitude and phase to be measured (e.g., RHOH and PHIH). The simultaneous H+V transmission and dual-channel reception is sometimes called the STAR mode (simultaneous transmit and receive). This allows the co–pol measurements to be made (ZDR, RHOHV, PHIDP and KDP). The alternating transmission dual-channel receiver allows both the co–pol and the cross-pol measurements to be made, i.e., it is the most complete.

Summary of Radar System Characteristics

The RVP8 supports all of these modes, but most polarization radar systems do not. As mentioned before, the measurement of cross-pol parameters such as LDR (fixed or alternating transmission and dual-channel reception) requires a radar system that has been optimized for cross–pol isolation, e.g., an offset feed antenna and no radome. By removing the feed, support struts and radome from the path of the radiation, the cross–pol isolation can be improved.

The single-channel alternating method has been used in several polarization radars for ZDR measurement. The advantage of this approach is that it is relatively easy to modify a conventional radar by simply adding a dual port feed and a high-power fast switch above the antenna rotary joints. The disadvantage is that the switch is costly and will eventually fail.

For these reasons, the STAR mode has come into recent use. No switch is required and the components are fairly reliable. The disadvantage of the approach (as it is usually implemented) is that a dual rotary joint and dual waveguides are required to duct both the H and the V through the antenna pedestal up to the antenna feed. In spite of this, the STAR mode offers perhaps the best approach for upgrading an existing radar or for factory installation on a new radar of conventional design.

B.3 RVP8 Dual-Channel Receiver Approach

Dual IFD and Rx Card

For dual-dual channel receivers, the RVP8 uses 2 IFD's and 2 RVP8/Rx cards. An example configuration is described and illustrated in Section 1.1.

Reference Clock to IFD

It is critical that both IFD's be phase-locked to a common reference clock. This clock, or a derivative frequency of the clock such as a COHO frequency, is input into each IFD to provide an absolute phase reference.

The RVP8 IFD phase-locks its sampling crystal to the reference clock input. Trigger generation by the RVP8 will also be phase locked to the reference clock. The reference clock must be in the range 2 to 60 MHz at 0 to –10 dBm and stable in phase to 10^{-7}. The RVP8 IFD must be specially configured with a locking crystal to enable this feature. SIGMET will either factory install the modification or assist the customer in performing the modification and supply the necessary components. See Section 2.2.12 for more information on installing and configuring to use an external reference clock.

SIGMET Antenna Mounted Receiver (AMR)

The SIGMET AMR converts a single polarization to a dual polarization system. It can be used on new radars or field upgrades of older systems. The AMR supports both STAR mode and LDR mode for H-only transmit. In the AMR, the RVP8 and other receiver components are packaged in a box that is mounted above the elevation axis of the antenna. Communication to/from the box is accomplished using a wireless LAN. For more information on the AMR, please see the AMR Technical Descritpion at www.sigmet.com.

B.4 Overview of Processing Algorithms

The RVP8 supports four polarization modes summarized in the table below. For each case, the standard moments (T, Z, V and W) are calculated as well. The notation for the outputs used here is similar to that in standard usage (e.g., Doviak and Zrnic). However, for LDR we use the notation LDRH to indicate that this is the LDR for horizontal transmission. The notation RHOH and PHIH is used to indicate the magnitude and phase of the covariance between the co- and cross–polarized channels for H transmit.

Table B–2: Supported Polarization Modes and Outputs

Case	Transmit	Receive	Processing Mode	Polarization Outputs
1	Fixed Horizontal or Fixed Vertical	Dual-Channel	PPP only	LDRH RHOH PHIH or LDRV RHOV PHIV
2	Simultaneous H+V (STAR Mode)	Dual-Channel	PPP or ZDR for FFT, Random Phase and DPRT1&2	ZDR PHIDP KDP RHOHV
3	Alternating H/V	Single-Channel	PPP only	ZDR PHIDP KDP RHOHV
4	Alternating H/V	Dual-Channel	PPP only	LDRH RHOH PHIH LDRV RHOV PHIV ZDR PHIDP KDP RHOHV

Input Receiver Sample Notation

For the discussion of polarization, we will adopt the notation used by Doviak and Zrnic. The received signal for pulse n from a single range bin shall be denoted as:

s^n_{hh}	Receive h: Transmit h	Horizontal co-polar signal
s^n_{vh}	Receive v: Transmit h	Horizontal cross–polar signal
s^n_{vv}	Receive v: Transmit v	Vertical co–polar signal
s^n_{hv}	Receive h: Transmit v	Vertical cross–polar signal

The pulse index is now indicated by the superscript as opposed to the subscript. The first subscript indicates the received polarization while the second subscript indicates the transmit polarization. If the transmit is the same as the received polarization, then this is called the co–polarized signal. If the transmit and receive are different then this is called the cross–polarized signal.

These variables are complex and are the same is the "s_n" notation used earlier, for example we can write:

$$s^n_{hh} = I^n_{hh} + j\ Q^n_{hh}$$

to show the relationship to the received I and Q values. Either filtered and unfiltered versions of the samples can be selected for processing. However, for convenience we will drop the s' notation for filtered samples.

Notation and Model for Correlations

The pulse pair processing mode is used for all of the polarization calculations, except that ZDR-only processing for the STAR case can be done in either FFT or random phase as well as pulse pair. As with the standard moments, the autocorrelations form the basis for the processing of the polarization variables.

The autocorrelations are computed in a manner identical to the standard moments, e.g., in pulse pair mode, the autocorrelations for the horizontal transmit co-polar channel are:

$$T_{o\,hh} = \frac{1}{M} \sum_{n=1}^{M} s_{hh}^{n}{}^{*} \; s_{hh}^{n}$$

$$R_{o\,hh} = \frac{1}{M} \sum_{n=1}^{M} s'^{n}_{hh}{}^{*} \; s'^{n}_{hh}$$

$$R_{1\,hh} = \frac{1}{M-1} \sum_{n=1}^{M-1} s'^{n}_{hh}{}^{*} \; s'^{n+1}_{hh}$$

$$R_{2\,hh} = \frac{1}{M-2} \sum_{n=1}^{M-2} s'^{n}_{hh}{}^{*} \; s'^{n+2}_{hh}$$

What is different is that for polarization systems, this processing can be applied in up to four separate channels (s_{hh}, s_{vh}, s_{vv} and s_{hv}). The physical model for the channel powers is identical to the model used for the standard moment cases, i.e.,

Co-Channel Power	**Cross-Channel Power**
$R_o^{hh} = g_h^r g_h^t \; S_{hh} + N_h$	$R_o^{vh} = g_v^r g_h^t \; S_{vh} + N_v$
$R_o^{vv} = g_v^r g_v^t \; S_{vv} + N_v$	$R_o^{hv} = g_h^r g_v^t \; S_{hv} + N_h$

Here S is used to denote the actual backscatter average power to the radar which, when multiplied by the appropriate transmitter and receiver gains, yields the actual measured power. Sometimes in comparing powers in two channels (e.g., ZDR and LDR) we will need to know the relative gains of the two channels. However, in many calculations, the relative gains cancel-out and in these cases the algorithms are implemented assuming all the gains are equal to 1.

In the algorithm descriptions, we will often use the notation common in the literature that (for example):

$$R_{o\,hh} = \frac{1}{M} \sum_{n=1}^{M} s'^{n}_{hh}{}^{*} \; s'^{n}_{hh} = \; < |s'_{hh}|^2 >$$

Noise Bias of Channel Power and Optional Correction

The average noise powers N_v and N_h are assumed to be receiver noise only. These bias the autocorrelations at lag zero, i.e., the channel power measurements. Autocorrelations at lags 1 and 2 are not biased by noise. Cross channel correlations are also not biased by noise, assuming that the noise in the two channels is independent (a good assumption).

The channel noise values are measured directly by the RVP8 during noise sampling. Whether to use these measured values to correct for the noise power when computing a channel power is optionally configured in the TTY setups. The choice is made in the mp TTY setup question "Polarization Parameters NoiseCorrected:YES/NO". If enabled, every time that a channel power is calculated, the noise power is subtracted.

This has some interesting effects. With no noise correction, ZDR values in weak signal regions will be biased by noise toward 0 dB (equal power), while if noise correction is enabled the values will be unbiased but will show substantial deviation over the region. The choice is up to the user.

Clutter Filtering

Clutter filtering is not currently supported in the dual polarization mode.

B.5 Case 1: Fixed Transmit: Dual-Channel Receiver

Input Receiver Samples

In fixed mode the radar is configured (either permanently or by means of a switch) to transmit either vertical or horizontal polarization with dual-channel reception of both the co- and cross–channel polarizations, e.g., transmit horizontal and receive both horizontal (co) and vertical (cross) polarizations.

The received samples in the two transmit cases are:

Transmit Horizontal or **Transmit Vertical**

$$\left[s_{hh}^1 : s_{vh}^1\right] \left[s_{hh}^2 : s_{vh}^2\right] \left[s_{hh}^3 : s_{vh}^3\right] \; . \; . \; . \; \left[s_{hh}^M : s_{vh}^M\right] \qquad \left[s_{vv}^1 : s_{hv}^1\right] \left[s_{vv}^2 : s_{hv}^2\right] \left[s_{vv}^3 : s_{hv}^3\right] \; . \; . \; . \; \left[s_{vv}^M : s_{hv}^M\right]$$

Calculation of the Polarization Measurands

The processing in this mode is done by pulse pair algorithm. The user may select a clutter filter, but in general this is not recommended for polarization studies since the clutter filter might interfere with the accuracy of sensitive parameters such as LDR.

The polarization measurands for the two transmit cases are as follows:

Transmit Horizontal or **Transmit Vertical**

$$LDRH = 10 \; LOG \left[\frac{S_{vh}}{S_{hh}}\right] \quad \text{or} \quad LDRV = 10 \; LOG \left[\frac{S_{hv}}{S_{vv}}\right]$$

$$= 10 \; LOG \left[\frac{< |s_{vh}|^2 > - N_v}{< |s_{hh}|^2 > - N_h}\right] - XDR \quad \text{or} \quad = 10 \; LOG \left[\frac{< |s_{hv}|^2 > - N_h}{< |s_{vv}|^2 > - N_v}\right] + XDR$$

$$RHOH = |\rho_h| \quad \text{or} \quad RHOV = |\rho_v|$$

$$PHIH = \arg[\rho_h] \quad \text{or} \quad PHIV = \arg[\rho_v]$$

Here, the H and V average channel powers are computed as follows with optional noise correction, i.e.,

Co- $\quad g_h^r g_h^t \; S_{hh} = < |s_{hh}|^2 > - N_h \quad$ or $\quad g_v^r g_v^t \; S_{vv} = < |s_{vv}|^2 > - N_v$

Cross- $\quad g_v^r g_h^t \; S_{vh} = < |s_{vh}|^2 > - N_v \quad$ or $\quad g_h^r g_v^t \; S_{hv} = < |s_{hv}|^2 > - N_h$

The complex covariance ρ (used above) is:

for H transmit $\quad \rho_h = \dfrac{< s_{vh} \; s_{hh}^* >}{\sqrt{S_{vh} S_{hh}}} \quad$ or for V transmit $\quad \rho_v = \dfrac{< s_{hv} \; s_{vv}^* >}{\sqrt{S_{hv} S_{vv}}}$

Fortunately, the algorithms do not require us to know all of the individual gain terms. They cancel in the calculation of ρ so are taken as =1 in the implementation. However, the differential receiver gain XDR must be known from calibration to calculate LDR:

$$dB \; Value \; is \quad XDR = 10 \; LOG \; xdr \quad where \; the \; linear \; value \; is \quad xdr = \frac{g_v^r}{g_h^r}$$

B.6 Case 2: Simultaneous Dual Transmit and Receive (STAR mode)

Input Receiver Samples

In this mode there is simultaneous transmit and receive of both vertical and horizontal polarization. For each pulse there is a measurement of the complex amplitude in each channel, i.e.,

$$\left[s_{hh}^{1} : s_{vv}^{1}\right] \left[s_{hh}^{2} : s_{vv}^{2}\right] \left[s_{hh}^{3} : s_{vv}^{3}\right] \; . \; . \; . \; \left[s_{hh}^{M} : s_{vv}^{M}\right]$$

We will assume that M samples are collected for processing, i.e., Note that even though there is cross–polarized radiation received in each channel, this cross–polar contribution can be neglected since the co-polarized received signal is much stronger.

Calculation of the Polarization Measurands

The processing in this case is done by pulse pair mode. However both FFT and random phase processing can be performed if only ZDR and standard moments are requested for ouput. In any mode, the user may select a clutter filter, but in general this is not recommended for polarization measurements since the clutter filter might interfere with the accuracy of sensitive parameters such as ZDR.

The RVP8 calculates the following polarization parameters:

$$ZDR = 10 \; LOG \left[\frac{S_{hh}}{S_{vv}}\right]$$

$$ZDR = 10 \; LOG \left[\frac{< \; |s_{hh}|^2 > \; - N_h}{< \; |s_{vv}|^2 > \; - N_v}\right] + GDR$$

$$RHOHV = |\rho_{hv}(0)|$$

$$PHIDP = \arg[\rho_{hv}(0)]$$

KDP based on least squares fit to PHIDP (see Section B.9).

where the following definitions are used:

$$g_h^r g_h^t \; S_{hh} = < \; |s_{hh}|^2 > \; - N_h \qquad g_v^r g_v^t \; S_{vv} = < \; |s_{vv}|^2 > \; - N_v$$

The noise powers the two channels are denoted as N_h and N_v . The noise corrections to S_{hh} and S_{vv} are optionally configured in the TTY setups. GDR is the total (transmit and receive) differential channel gain. It must be calibrated for the system.

$$dB \; Value \; is \quad GDR = 10 \; LOG \; xdr \quad where \; the \; linear \; value \; is \quad gdr = \frac{g_v^r g_v^t}{g_h^r g_h^t}$$

The correlation function is computed from:

$$\rho_{hv}(0) = \frac{< s_{vv} \; s_{hh}^{*} >}{\sqrt{S_{hh} S_{vv}}}$$

The gain terms cancel in the calculation of ρ so in the implementation they are simply assumed to be =1.

B.7 Case 3: Alternating H/V Transmit: Single-Channel Receiver

Input Receiver Samples

This is the traditional ZDR radar with a high-power fast switch that alternates between horizontal and vertical on each pulse. The switch is made just prior to the transmit pulse so that the transmitter radiates and then receives at a single polarization for each pulse. Thus the samples are:

$$s_{hh}^{1} \qquad s_{vv}^{2} \qquad s_{hh}^{3} \quad \ldots \quad s_{vv}^{M+1}$$

For the discussion below we will assume that there are M+1 total samples with M/2 horizontal pulses indexed by (1, 2, 3...M–1) and M/2+1 vertical pulses indexed at (2, 4, 6, ...M). Note that the processor does not assume that the first pulse in a sequence is horizontal.

Calculation of the Polarization Measurands

The processing is done in pulse pair with optional clutter filter. Again, for accurate ZDR measurements, the clutter filter may interfere.

The RVP8 calculates the following:

$$ZDR = 10\ LOG\left[\frac{S_{hh}}{S_{vv}}\right]$$

$$ZDR = 10\ LOG\left[\frac{<|s_{hh}|^2> - N_h}{<|s_{vv}|^2> - N_v}\right] + GDR$$

$$PHIDP = \frac{1}{2}\ \arg\left[R_a\ R_b^*\right]$$

$$RHOHV = \frac{|\rho_{hv}(T_s)|}{\left[\rho_{hv}(2T_s)\right]^{0.25}}$$

KDP based on least squares fit to PHIDP (see Section B.9).

where the following definitions are used:

$$|\rho_{hv}(T_s)| = \frac{|R_a| + |R_b|}{2\sqrt{S_{hh}S_{vv}}}$$

$$\rho(2T_s) = \frac{\left|\sum_{n=1}^{M/2-1}(s_{hh}^*[2n-1]\ s_{hh}[2n+1] + s_{vv}^*[2n]\ s_{vv}[2n+2])\right|}{(M/2-1)(S_{hh}+S_{vv})}$$

$$R_a = \frac{1}{M/2}\sum_{n=1}^{M/2} s_{hh}^{2n-1}*\ s_{vv}^{2n} \qquad \text{and} \qquad R_b = \frac{1}{M/2}\sum_{n=1}^{M/2} s_{vv}^{2n}*\ s_{hh}^{2n+1}$$

The calculation of the channel powers ($<|s_{hh}|^2>$ and $<|s_{vv}|^2>$) is done using alternating pulses in this case. Note that in the calculation of R_b, the RVP8 uses the extra M+1 sample. The gain terms cancel in the calculation of ρ so in the implementation they are simply assumed to be =1.

B.8 Case 4: Alternating H/V Transmit: Dual-Channel Receiver

Input Receiver Samples

This is the most comprehensive case of polarization operation since it permits calculation of all of the polarization measurands. In this case the transmitter alternates pulse-to-pulse between horizontal and vertical polarization and the dual-channel receiver provides measurement of both the co- and the cross-polarized return, i.e.,

$$\left[s_{hh}^{1} : s_{vh}^{1}\right] \left[s_{vv}^{2} : s_{hv}^{2}\right] \left[s_{hh}^{3} : s_{vh}^{3}\right] \left[s_{vv}^{3} : s_{hv}^{3}\right] \ . \ . \ . \ \left[s_{vv}^{M+1} : s_{hv}^{M+1}\right]$$

We will assume that M+1 samples are collected for processing (an extra sample is required for the calculation Rb per section B.7).

Calculation of the Polarization Measurands

The RVP8 calculates the following:

Co-polar channel measurements

ZDR, PHIDP, RHOHV	Identical to alternating case Section B.7.

Cross-polar channel measurements

LDRH, RHOH, PHIH LDRV, RHOV, PHIV	Identical to fixed case Section B.5.

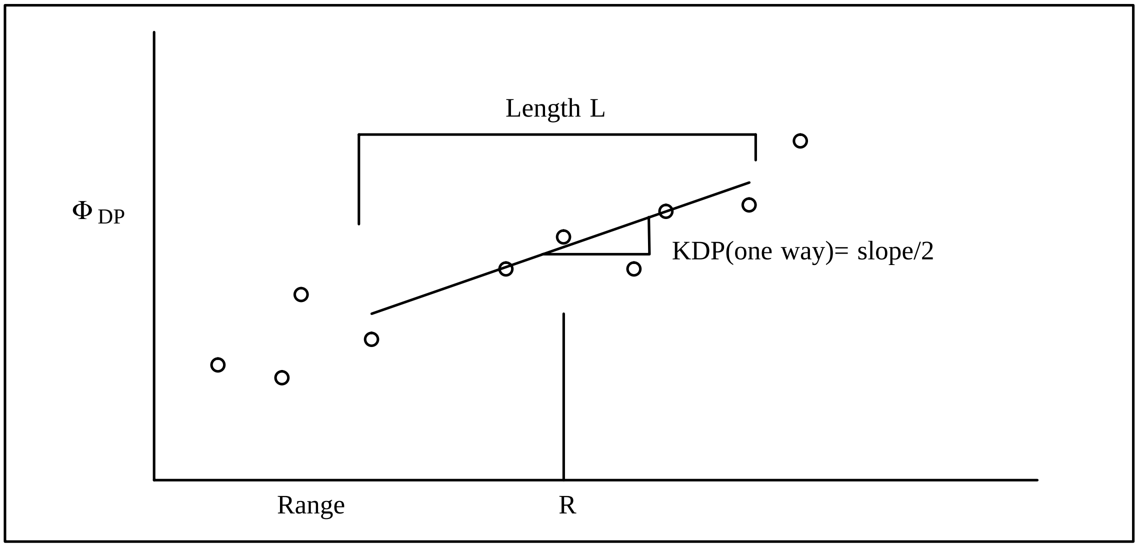
The co-polar channel measurements are exactly as they are for the alternating single-receiver case. The cross–polar measurements are calculated using fixed case algorithms except they are calculated for *both* H and V polarizations.

B.9 KDP Calculation

In all modes that compute PHIDP, the signal processor can also be configured to compute KDP- the specific differential phase in units of degrees per km. This is the range derivative of PHIDP. There are two techniques that have been used to obtain this:

- The smoothed range derivative.
- The slope from a least squares fit.

Figure B–1: Least Squares KDP calculation



The graph shows the thresholded differential phase vs range. This is the starting point for the algorithm. The length scale L is selectable by the user in the TTY setups (mp section, KDP Length in km, default 5.00 km). The KDP value for a bin at range R is computed from a least squares fit that includes points that are within +–L/2 as indicated in the figure. PHIDP is output by the processor on the unambiguous interval of 0 to 180 degrees. Before fitting, the points are first unfolded to a common interval by starting at the left-most point and then moving right assuming that a difference of more than 1/2 the unambiguous interval is the result of folding. Since it is the slope that is of interest, the absolute interval is not critical, as long as the points are in a common interval.

After fitting, the slope is obtained which corresponds to the 2-way KDP since it is based on the 2-way measurement of PHIDP . To be consistent with most values in the literature, the slope value is divided by 2 so that the final output is the one-way KDP in degrees per km (with a wavelength scaling in the data format).

This procedure is repeated for each bin. Thus if the bin spacing is 250 m, the output bin spacing of KDP will be 250 m. It is required that there be at least 50% of the possible number of bins present in the interval L to calculate a valid KDP, else the KDP is set to the threshold value. Since the input PHIDP values are already thresholded, the only additional threshold on KDP is this 50% rule.

B.10 Standard Moment Calculations (T, Z, V, W)

Overview

Standard moments are available for all four of the polarization cases. Since there can be up to four different channels of time series input, there are several choices for computing the standard moments. For example, in the STAR mode (Case 2), the standard moments can be computed from:

- s_{hh} samples
- s_{vv} samples
- Average of the results from the s_{hh} and s_{vv} samples

The third case is handled by averaging the individual channel correlations, and then using the average correlations in the standard moment processing. The averaging must take into account the differential gain of the channels.

The selection of which method to use is made in setup. There are four questions posed in the mp section:

```
T/Z/V/W computed from:   H–Xmt:YES   V–Xmt:YES
T/Z/V/W computed from:   Co–Rcv:YES  Cx–Rcv:NO
```

The first two questions are used to specify that ***given a choice*** between vertical and horizontal transmit, which transmit polarization to use. Thus for the fixed H or V case where there is only one transmit polarization, this question does not apply. The processor will simply use samples for the polarization is transmitted.

The second two questions are used to specify that ***given a choice*** between using the co- or cross-polar receivers which one shall be used. This question applies only to systems that can measure LDR, i.e., fixed or alternating transmit, dual-channel receiver systems).

The tables in the sections below summarize the standard moment calculations for each of the four modes and how to configure the four TTY setup responses. Note that these are the only supported modes. Some combinations of responses are unsupported. For example, it is not supported to answer both Co-Rcv: NO and Cx-Rcv: NO.

The top of each table identifies the transmitter/receiver case and what samples are available. The notation HH signifies that the s_{hh} samples are available. The tables use "—–" to indicate that either a YES or NO response will cause the same result, i.e., the RVP does not care what response is made. In cases where averaging is performed, the type of weighting used is indicated (either GDR or XDR weighting).

Model for standard moment autocorrelations

The model for the moment autocorrelation calculations is as follows (using R_o as an example):

$$R_0^{hh} = g_h^r\, g_h^t\, S_{hh} + \overline{N}_h \qquad R_0^{vh} = g_v^r\, g_h^t\, S_{vh} + \overline{N}_v$$

$$R_0^{vv} = g_v^r\, g_v^t\, S_{vv} + \overline{N}_v \qquad R_0^{hv} = g_h^r\, g_v^t\, S_{hv} + \overline{N}_h$$

where:

$R_0^{hh},\ R_0^{vh},\ R_0^{vv},\ R_0^{hv}$	Are the autocorrelations if the samples at lag zero.
$S_{hh},\ S_{vh},\ S_{vv},\ S_{hv}$	The average power returned from the scatterers.
$g_h^r,\ g_v^r$	Receiver gains for horizontal and vertical receive.
$g_h^t,\ g_v^t$	Transmitter gains for horizontal and vertical transmit.
$\overline{N}_h,\ \overline{N}_v$	Measured noise power of the samples.

In other words, the power that is measured in a channel has two components:

- Backscattered power from the targets that is effected by the transmitter and receiver channel gains.
- Receiver noise which is measured by the RVP8 during noise sampling.

In the case of R1 and R2 autocorrelations, the model is similar except that there is no noise bias.

Calibration Parameters

For dBZ calculations, a calibration constant is required, i.e., the dBZ_o value in Section 5.4. Depending on the polarization case and the technique selected for standard moment calculation, it may also be required to have GDR and XDR, i.e.,

- GDR- The ratio of the total gains (transmit/receive) of the two co–receive channels.
- XDR- The ratio of the receiver gains in a dual receiver system. This is not required for the Case 2: STAR or the Case 3: Alternating Single–Channel.

The RVP8 supports a single calibration reflectivity dBZ_o. In all cases it is assumed that the dBZ_o is for the horizontal co–receive (HH) channel. The only exception is for fixed vertical polarization, in which the algorithm assumes that the calibration is for the vertical co-receive (VV) channel. XDR and GDR are also downloaded and used to adjust the dBZ_o as required depending on the user's selection for the standard moments. For example, in STAR mode, if the user selects dBZ to be computed from the VV channel, the dBZ_o for the HH and a GDR adjustment are used to calculate the dBZ in the VV channel.

The remainder of this section discusses the standard moment calculation options for the each polarization case. For a discussion of how to calibrate XDR and GDR see Section B.12.

Case 1H: Fixed Horizontal Transmit, Dual Channel Receive- (HH, VH)8				
dBZo from HH Channel	**TTY Setup Question Responses**			
Calculate T, Z, V, W from:	**H–Xmt**	**V–Xmt**	**Co–Rcv**	**Cx–Rcv**
HH (co) (Recommended)	—	—	YES	NO
VH (xdr^{-1} weighting)	—	—	NO	YES
HH+VH (xdr^{-1} weighting)	—	—	YES	YES

HH Channel (co–pol)

This is the recommended channel for the case of linear polarization. The reason is that for linear polarization, the co–polar channel will have the strongest signal. Processing is identical to a conventional radar.

VH Channel (cross–pol)

This choice would be used for circular or elliptic transmit polarization. Since the algorithm assumes that dBZo is from the co–polar channel, xdr is used to adjust the autocorrelations as follows:

$$T_0 = xdr^{-1}\ T_o^{vh}$$
$$R_0 = xdr^{-1}\ R_o^{vh}$$
$$R_1 = xdr^{-1}\ R_1^{vh}$$
$$R_2 = xdr^{-1}\ R_2^{vh}$$
$$N = xdr^{-1}\ N_v$$

These adjusted autocorrelations are then used as per the standard moment processing for a conventional radar. To illustrate this, consider the example of reflectivity processing. The radar equation can be written as (see section 5.2.6):

$$Z^{vh} = C\ S_{vh}\ r^2 = \left[\frac{Cr_0^2 N_v}{g_v^r g_h^t}\right]\left[\frac{r^2}{r_o^2}\right]\left[\frac{T_o^{vh} - N_v}{N_v}\right], \text{where } T_0^{vh} = g_v^r g_h^t S_{vh} - N_v$$
$$= \left[\frac{Cr_0^2 N_h}{g_h^r g_h^t}\right]\left[\frac{r^2}{r_o^2}\right]\left[\frac{g_h^r}{g_v^r}\right]\left[\frac{T_o^{vh} - N_v}{N_h}\right]$$

The third term is simply 1/XDR so that we can write:

$$Z^{vh} = \left[\frac{Cr_0^2 N_h}{g_h^r g_h^t}\right]\left[\frac{r^2}{r_o^2}\right]\left[\frac{xdr^{-1}T_o^{vh} - xdr^{-1}N_v}{N_h}\right]$$

In this case, the first term is the dBZ_o for the HH channel. Thus we can use the dBZ_o for the HH channel to calibrate the cross-channel, if we first adjust the cross–channel noise and power by 1/xdr and then normalize by N_h. The reflectivity calculation assumes that the calibrated xdr value compensates for any differences in the radar constant between the two channels, i.e., we do not need to have separate radar constants for the two channels.

HH+VH Channels

This choice would be used for elliptic transmit polarizations that give comparable return signal in both the co- and cross-channels. The approach is to obtain average autocorrelation functions as follows:

$$T_0 = \frac{T_o^{hh} + xdr^{-1}\ T_o^{vh}}{2}$$

$$R_0 = \frac{R_o^{hh} + xdr^{-1}\ R_o^{vh}}{2}$$

$$R_1 = \frac{R_1^{hh} + xdr^{-1}\ R_1^{vh}}{2}$$

$$R_2 = \frac{R_2^{hh} + xdr^{-1}\ R_2^{vh}}{2}$$

$$N = \frac{N_h + xdr^{-1}\ N_v}{2}$$

These adjusted autocorrelations are then used as per the standard moment processing for calibration with respect to the HH channel.

Case 1V: Fixed Vertical Transmit and Dual Channel Receive- (VV, HV)				
dBZo from VV Channel	**TTY Setup Question Responses**			
Calculate T, Z, V, W from:	**H–Xmt**	**V–Xmt**	**Co–Rcv**	**Cx–Rcv**
VV (co)	——	——	YES	NO
HV (xdr weighting)	——	——	NO	YES
VV+HV (xdr weighting)	——	——	YES	YES

This is the only case for which the calibration constant dBZ_o for the VV channel should be downloaded to the signal processor.

VV Channel (co–pol)

This is the recommended channel for the case of linear polarization. The reason is that for linear polarization, the co-polar channel will have the strongest signal. Processing is identical to a conventional radar.

HV Channel (cross–pol)

This choice would be used for circular or elliptic transmit polarization when most of the return is in the cross-pol channel. Since the algorithm assumes that dBZo is from the co-polar channel, xdr is used to adjust the autocorrelations as follows:

$$T_0 = xdr\ T_o^{hv}$$

$$R_0 = xdr\ R_o^{hv}$$

$$R_1 = xdr\ R_1^{hv}$$

$$R_2 = xdr\ R_2^{hv}$$

$$N = xdr\ N_h$$

These adjusted autocorrelations are then used as per the standard moment processing with dBZo calibrated with respect to the VV channel.

VV+HV Channels

This choice would be used for elliptic transmit polarizations that give comparable return signal in both the co- and cross-channels. The approach is to obtain average autocorrelation functions as follows:

$$T_0 = \frac{T_o^{vv} + xdr\ T_o^{hv}}{2}$$

$$R_0 = \frac{R_o^{vv} + xdr\ R_o^{hv}}{2}$$

$$R_1 = \frac{R_1^{vv} + xdr\ R_1^{hv}}{2}$$

$$R_2 = \frac{R_2^{vv} + xdr\ R_2^{hv}}{2}$$

$$N = \frac{N_v + xdr\ N_h}{2}$$

These adjusted autocorrelations are then used as input to the standard moment processing algorithms with dBZ_o calibrated with respect to the VV channel.

Case 2: Simultaneous Transmit and Receive- STAR (HH, VV) Case 3: Alternating Transmit Single-Channel Receive (HH, VV)				
dBZo from HH Channel	**TTY Setup Question Responses**			
Calculate T, Z, V, W from:	**H–Xmt**	**V–Xmt**	**Co–Rcv**	**Cx–Rcv**
HH	YES	NO	—	—
VV (gdr^{-1} weighting)	NO	YES	—	—
HH+VV (gdr^{-1} weighting)	YES	YES	—	—

A fundamental difference between these two cases is that for all standard moment processing choices, the STAR case has double the number of samples as compared to the single-channel alternating case. However, the processing is otherwise identical.

HH Channel

Since the HH channel is directly calibrated this is the recommended choice. Processing is identical to a conventional radar.

VV Channel

In this case, GDR is used to adjust the autocorrelations as follows:

$$T_0 = gdr^{-1}\ T_o^{vv}$$

$$R_0 = gdr^{-1}\ R_o^{vv}$$

$$R_1 = gdr^{-1}\ R_1^{vv}$$

$$R_2 = gdr^{-1}\ R_2^{vv}$$

$$N = gdr^{-1}\ N_v$$

These adjusted autocorrelations are then used as input to the standard moment processing algorithms with dBZo calibrated with respect to the HH channel.

HH+VV Channels

This approach gives the benefit of doubling the number of samples used for the reflectivity calculation.

$$T_0 = \frac{T_o^{hh} + gdr^{-1}\ T_o^{vv}}{2}$$

$$R_0 = \frac{R_o^{hh} + gdr^{-1}\ R_o^{vv}}{2}$$

$$R_1 = \frac{R_1^{hh} + gdr^{-1}\ R_1^{vv}}{2}$$

$$R_2 = \frac{R_2^{hh} + gdr^{-1}\ R_2^{vv}}{2}$$

$$N = \frac{N_h + gdr^{-1}\ N_v}{2}$$

These adjusted autocorrelations are then used as input to the standard moment processing algorithms with dBZo calibrated with respect to the HH channel.

Case 4: Alternating Dual-Channel (HH, VH, VV, HV)				
dBZo from HH Channel	**TTY Setup Question Responses**			
Calculate T, Z, V, W from:	H–Xmt	V–Xmt	Co–Rcv	Cx–Rcv
HH	YES	NO	YES	NO
VH (xdr^{-1} weighting)	YES	NO	NO	YES
VV (gdr^{-1} weighting)	NO	YES	YES	NO
HV (xdr/gdr weighting)	NO	YES	NO	YES
HH+VV (gdr^{-1} weighting)	YES	YES	YES	NO
HV+VH (xdr & gdr weighting)	YES	YES	NO	YES

HH Channel

Since the HH channel is directly calibrated this is the recommended choice. Processing is identical to a conventional radar.

VH Channel

Processing is identical to Case 1H: Horizontal Transmit HV Processing.

VV Channel

Processing is identical to Cases 2&3:STAR and Single Channel Alternating VV Processing.

HV Channel

The weighting in this case uses both xdr and gdr.

$$T_0 = \frac{xdr}{gdr} T_o^{hv}$$

$$R_0 = \frac{xdr}{gdr} R_o^{hv}$$

$$R_1 = \frac{xdr}{gdr} R_1^{hv}$$

$$R_2 = \frac{xdr}{gdr} R_2^{hv}$$

$$N = \frac{xdr}{gdr} N_h$$

These adjusted autocorrelations are then used as input to the standard moment processing algorithms with dBZo calibrated with respect to the HH channel.

HH + VV Channels

Processing is identical to Cases 2&3: STAR and Single Channel Alternating HH+VV Processing.

HV + VH Processing

The weighting here has to correct for both transmitter and receiver effects in order to use the HH channel dBZ_o.

$$T_0 = \frac{\frac{xdr}{gdr} T_o^{hv} + xdr^{-1}\ T_o^{vh}}{2}$$

$$R_0 = \frac{\frac{xdr}{gdr} R_o^{hv} + xdr^{-1}\ R_o^{vh}}{2}$$

$$R_1 = \frac{\frac{xdr}{gdr} R_1^{hv} + xdr^{-1}\ R_1^{vh}}{2}$$

$$R_2 = \frac{\frac{xdr}{gdr} R_2^{hv} + xdr^{-1}\ R_2^{vh}}{2}$$

$$N = \frac{\frac{xdr}{gdr} N_h + xdr^{-1}\ N_v}{2}$$

These adjusted autocorrelations are then used as input to the standard moment processing algorithms with dBZo calibrated with respect to the HH channel.

An example of how this weighted averaging works is given here. Suppose that we want to compute the average of the reflectivities for the VH and HV channels,

$$Z^{hv+vh} = Cr^2\ \frac{S_{hv} + S_{vh}}{2}$$

$$= Cr^2\ \frac{\frac{T_0^{hv} - N_h}{g_h^r g_v^t} + \frac{T_o^{vh} - N_v}{g_v^r g_h^t}}{2} = \frac{Cr^2}{g_h^r g_h^t}\ \frac{(T_0^{hv} - N_h)\frac{g_h^t}{g_v^t} + (T_o^{vh} - N_v)\frac{g_h^r}{g_v^r}}{2}$$

but since $xdr = \frac{g_v^r}{g_v^h}$ and $gdr = \frac{g_v^r g_v^t}{g_h^r g_h^t}$

$$Z^{vh+hv} = \frac{Cr^2}{g_h^r g_h^t}\left[\frac{\frac{xdr}{gdr} T_0^{hv} + xdr^{-1} T_0^{vh}}{2} - \frac{\frac{xdr}{gdr} N_h + xdr^{-1} N_v}{2}\right]$$

$$Z^{vh+hv} = \frac{Cr^2}{g_h^r g_h^t}\ \left[T_0 - N\right] = \left[\frac{Cr_o^2 N_h}{g_h^r g_h^t}\right]\left[\frac{r^2}{r_o^2}\right]\left[\frac{T_0 - N}{N_h}\right]$$

The first term in brackets is precisely dBZ_o for the HH channel. Thus if we average the correlations using the appropriate GDR and xdr weighting as shown above, then the average reflectivity is obtained by using conventional processing with the HH channel dBZ_o.

B.11 Thresholding of Polarization Parameters

The thresholding of polarization parameters by the processor eliminates bins with weak or uncertain signals. Note that the thresholding can be disabled if it is desired to see all of the data regardless of the data quality.

All of the polarization parameters are based on power ratios. The RVP8 requires that each power term in a ratio pass a signal-to-noise test similar to the log power test. For example, there are up to four different powers that can be calculated (alternating dual-channel case) so the tests for each of these are:

$$\frac{< |s_{hh}|^2 >}{N_h} > N_{thresh}$$

$$\frac{< |s_{hv}|^2 >}{N_h} > N_{thresh}$$

$$\frac{< |s_{vv}|^2 >}{N_v} > N_{thresh}$$

$$\frac{< |s_{vh}|^2 >}{N_h} > N_{thresh}$$

where the linearized threshold that is input as the dB LOG threshold, i.e.,

$$N_{thresh} = 10^{LOGthresh/10}$$

For example, a valid LDRH requires both a valid S_{hh} and a valid S_{vh}. The parameters RHOH and PHIH have the same requirement since they are the magnitude and phase of the cross-correlation function which is based on S_{hh} and S_{vh}.

There are two exceptions:

ZDR

ZDR requires that both S_{hh} and S_{vv} pass the signal-to-noise tests noted above. However, ZDR can be additionally thresholded by any of the other threshold parameters (LOG, SIG, SQI, CSR) similar to a standard moment. See section 5.3 for a description of the standard moment thresholding.

PHIDP for single channel alternating case

PHIDP requires that both S_{hh} and S_{vv} pass the signal-to-noise tests noted above. In the single channel alternating case, PHIDP must also satisfy the additional test that the Doppler velocity at the range bin must be valid, i.e., not thresholded by its own criteria. This is because the algorithm for PHIDP in this case essentially subtracts the phase change due to the Doppler velocity. If the Doppler velocity is uncertain, the algorithm cannot produce reliable results.

B.12 Calibration Considerations

Polarization systems require additional calibration as compared to conventional systems. There are three aspects to the calibration:

- dBZ_o measurement in both channels for dBZ and dBT calibration.
- gdr measurement for ZDR calibration.
- xdr measurement for LDR calibration.

These are discussed below.

dBZ_o Calibration for dBZ

The RVP8 supports separate calibration of both polarization channels. Measurement of dBZ_o for each channel of a dual polarization system is identical to the conventional radar case described in Section 5.4. Note that for a single-channel switching system, the only difference between the horizontal and vertical signal paths occurs after the high power switch, i.e., differential insertion loss of the switch itself and any differential insertion loss of the waveguides and feed after the switch. This means that for single-channel switching systems it may be sufficient to calibrate at one polarization and then adjust the calibration of the other channel by the differential gain GDR (see below).

GDR Calibration for ZDR

GDR is the dB value of the relative gain between the co-polarized channels including both transmitter and receiver gain, i.e.,

$$GDR = 10\ LOG\ \frac{g_v^r g_v^t}{g_h^r g_h^t} \qquad \text{and} \qquad gdr = \frac{g_v^r g_v^t}{g_h^r g_h^t}$$

GDR is input into the processor as a dB value. However, for analyses in this chapter, the linear gdr value is sometimes more convenient.

In principle, if dBZ_o could be calibrated perfectly in both channels, measurement of GDR would not be required. In practice, this is not possible because dBZ_o cannot be calibrated to an absolute accuracy sufficient for ZDR, i.e., to 1/16th of a dB. Therefore, the RVP8 uses the GDR approach.

Since GDR includes both transmitter and receiver differential gains, accurate calibration requires that an actual target be observed. One way to do this is as follows:

- Set the GDR to be 0 dB using your application software (e.g., for SIGMET IRIS systems in the setup utility RVP section). Disable clutter filtering for ZDR in either your application software (by selecting filter 0) or explicitly in the RVP8 TTY setups mp section.
- Place the antenna at 90 degrees elevation (vertical incidence) during moderate to heavy rain. The melting layer should be at a height that is well above the recovery zone of the T/R and in the antenna "far zone". A melting layer higher than 2 km is suggested, but the specific characteristics of the radar should be considered.

- Collect ZDR data at vertical incidence while the antenna is rotating in azimuth.
- Use a separate application program to average the ZDR values around a full 360 degrees at each range bin (height). Generate a plot of 360-average ZDR vs height.
- You should observe that the average ZDR values in regions of strong signal (>20 dB SNR) below the bright band are approximately constant with height. This is the value that should be used in your application software for GDR.
- Enter the value and repeat the calibration to verify that the average ZDR is now 0 dB.

The rationale for this approach is as follows. When viewed at vertical incidence, rain should have a ZDR of 0 dB since the drops will all appear circular. The reason for averaging over 360 degrees is to cancel-out effects from sidelobe contamination from nearby ground targets and other artifacts of the antenna/feed/radome system. For example the radome may have an obstruction light on the top. Some of these artifacts can be minimized by assuring the the weather targets are strong, i.e., heavy rain is preferred for this calibration.

XDR Calibration for LDR

XDR is the dB value of the relative gain between the co- and cross-receiver channels for LDR measurements. Analogous to GDR, it is defined as the dB value of the ratio of the vertical to horizontal receiver gains, i.e.,

$$XDR = 10\ LOG\ \frac{g_v^r}{g_h^r} \qquad \text{and} \qquad xdr = \frac{g_v^r}{g_h^r}$$

Three techniques for calibration of XDR are discussed. It is recommended for the transmitter to be off for all of these methods.

- 1. **Solar method**
  Use the sun to measure LDR. The measured value of LDR is then the XDR offset. LDR should be measured in fixed mode for both LDRH and LDRV. The values should be reciprocal (e.g., +1 dB and –1 dB). Use the average of the absolute value if they are not precisely reciprocal (e.g., for +1.4 and –1.2 use 1.3). Finally after inputting the XDR value, retest to verify that the sun has been properly corrected to have zero LDR.
- 2. **Signal generator method with connection to waveguide**
  Connect a signal generator with a splitter to both channels and measure XDR directly. This does not account for any effects that are before the coupler (e.g., waveguide, feed, radome, antenna gain).
- 3. **Linear feed horn remote radiator method**
  Use a calibrated linear feed horn with an RF source located several hundred meters from the radar. Maximize the H channel return and measure the response using the RVP8 pr command “Filtered” power in the “Primary Channel”. Now rotate the feed horn to vertical and maximize the power in the “Secondary Channel”. The difference in dB is XDR. Note that signal multi–path effects could bias the results from this technique.

In all cases it is recommended that for the calibration, XDR be set to 0 dB in the application user software and that the RVP8 TTY setups be configured as follows:

- Noise correction enabled for LDR and noise sample taken prior to the measurements (with care not to sample with a test signal turned–on or while looking at the sun).
- Clutter correction disabled for LDR.